A Theory of Hung Jury and Informative Voting

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Abstract

This paper investigates a jury decision when hung juries and retrials are possible. When jurors in subsequent trials know that previous trials resulted in hung juries, informative voting can be an equilibrium for some utility parameters if and only if the accuracy of signals for innocence and guilt are exactly identical. Moreover, if jurors are informed of numerical split of votes in previous trials, then informative voting is not an equilibrium regardless of signal accuracy. Thus the claim of Coughlan (2000) that mistrials facilitate informative voting holds only in limited circumstances. Also, the analysis gives support to measures such as jury selection and sequestering, which try to prevent jurors from accessing information on previous trials.

1 Introduction

In many jurisdictions in the United States and elsewhere, unanimity among jurors is required for jury verdict. The unanimity rule is commonly believed to minimize the possibility of convicting an innocent defendant at the cost of

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increasing the possibility of acquitting a guilty one. This view is challenged by Feddersen and Pesendorfer (1998) in light of game-theoretic analysis of voting behavior. They note that jurors rarely have incentives to vote informatively, that is, it may not be in the interest of the juror to cast a vote according to her impression of guilt or innocence of the defendant. They show that, if jurors vote strategically, the unanimity rule may convict the innocent and acquit the guilty more often than most rules, including the simple majority rule.

Coughlan (2000) provides a counterargument to their claim in his study of jury decision with mistrials. He points out that unanimity is required for either conviction or acquittal in many jurisdictions. Otherwise a hung jury occurs and the case faces a new trial in the future with a new group of jurors. He considers a reduced model in which jurors in a subsequent trial possess the same belief on the defendant’s guilt as those in previous trials, and argues that informative voting is an equilibrium for a nontrivial range of parameters.

This paper explicitly models a dynamic game of jury decision with hung juries. In particular, we allow jurors in subsequent trials know the fact that the case resulted in a mistrial in previous trials and use this fact to infer the likelihood of guilt or innocence of the defendant. This informational aspect of the dynamic jury procedure is not taken into account in Coughlan (2000), but our analysis shows that it affects behavior of jurors in a crucial manner.

In the fully dynamic setting, we find that informative voting is very unlikely to obtain if information is transmitted from previous trials. More specifically, we first consider an environment in which jurors know the existence of hung juries that occurred previously (if any). We show that informative voting is an equilibrium for some utility parameter if the accuracy of signals for innocence and guilt are exactly identical. As the equivalence of signal accuracies is assumed in Coughlan (2000), we confirm his analysis in this particular case. Then more importantly, we show that if the signal accuracies of guilt and innocence are not exactly identical, then informative voting can never be an equilibrium. Moreover, if jurors are informed of numerical split of votes in previous trials, then informative voting is not an equilibrium regardless of signal accuracy.

Intuition of the above results are as follows. Suppose that the signal of guilt is more accurate than that of innocence. Then, jurors who vote informatively are more likely to disagree when the defendant is innocent.

1A mistrial may be declared for a number of other reasons, such as juror misconduct. However we focus on a mistrial that occurs because of a hung jury and use these terms interchangeably.
than when she is guilty. Intuitively, this is because an innocent defendant may still be seen as guilty because of a mistaken signal and votes tend to be split.\footnote{A formal analysis is more complicated. In particular, a noisy signal increases the probability that the jury agrees on the wrong decision. Our formal analysis, especially Lemma 1, shows that the total effect is that a noisier signal produces hung juries with higher probability.} Hence, if a juror knows that there was a hung jury before the current trial, she infers that the defendant is more likely to be innocent by Bayes’ law. If hung juries occur repeatedly, information from previous trials will become so strong that a juror is willing to ignore her own private impression of guilt or innocence and vote to acquit. Thus informative voting fails to be an equilibrium.

A number of implications are drawn from our formal analysis. First, our results show that the informative voting cannot be expected to occur in a full-fledged dynamic model of voting with mistrials if information cannot be hidden from jurors. In the context of the debate on unanimity jury verdicts, a severe qualification needs to be added to the assertion that mistrials facilitate informative voting: Jurors should be prevented from knowing about previous trials, or the parameter of signal accuracy is nongeneric. Thus our paper complements the analysis of Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1998) who emphasize the importance of strategic voting in jury design.

Second, our analysis has implications on legal processes such as jury selection and sequestering. Jury selection is composed of several stages. In one of them, the so-called voir dire, the judge typically asks prospective jurors whether they have heard about the case, and an individual who answers positively are asked more questions such as whether she can disregard the outside information when deciding her vote. Depending on the answers, a prospective juror may be excluded from the jury. Also, jurors who are chosen are sometimes sequestered during deliberation, i.e., kept in a place so that they do not hear about the case from media such as newspapers and television. Despite such measures to prevent jurors from getting information, jurors are sometimes informed. For instance, a juror’s reading of newspaper articles pertaining to the trial may not be grounds for reversal, new trial, or mistrial unless the articles are of such a character that they might have resulted in prejudice to the losing party (Roy v. State, 213 Kan. 30, 32, 514 P.2d 832 (1973)). In fact, the granting of a mistrial on this ground seems to lie within the discretion of the trial court, although exact implimentation varies across jurisdictions (State v. Jakeway, 221 Kan. 142, 148, 558 P.2d 113 (1976)). For example, in State v. Baker (1980) 227 Kan 377, 607 P2d 61, jurors knew that there had been trials before from newspaper, and the
judge even told jurors that there had been a hung jury before. The judge said:

As some of you may have guessed this is the third time around for this case. The first case was sent back by the Supreme Court. The second case the jury could not agree. (Court’s remark to the jury in State v. Baker (1980) 227 Kan 377, 607 P2d 61)

Also, the court denied a request to sequester the jury made by the defendant in this case. In short, the current system in principle tries to keep jurors from knowing about previous trial, but the measures are not strictly enforced.

Our analysis gives certain justification of strict legal procedures that try to prevent jurors from knowing about previous trials. The results of this paper suggests that allowing information transmission between trials makes informative voting difficult to obtain in equilibrium.\(^3\) To the extent to which informative voting is desirable in the jury process, measures such as a strict jury selection process and wider use of sequestering may be an effective way to achieve this goal.

1.1 Relation with the Literature

One methodological contribution of this paper is the introduction of a fully dynamic analysis of voting behavior in a jury with mistrials. There are not many game-theoretic analysis of jury design when hung juries and retrials are possible.\(^4\) Coughlan (2000) forms a basis of the current paper. Neilson and Winter (2005) conduct numerical analysis of error probabilities in a similar model of hung juries under non-strategic voting. They invoke Coughlan (2000) to justify their assumption of non-strategic voting. As we have pointed out, Coughlan (2000) implicitly assumes that jurors have the same belief on guilt or innocence of the defendant no matter how many mistrials have occurred. Our analysis suggests that such an assumption is valid if and only if

\(^3\)As in Coughlan (2000), informative voting can be an equilibrium when information of previous trials are unavailable to jurors in subsequent juries. However, the condition for the conclusion is slightly altered from the original he obtains. This is because jurors who do not know how many hung juries occurred before them form a belief on the guilt of the defendant by Bayes’ law, and the resulting belief is in general different from the prior probability \(r\) of guilt. We just point out this fact and omit complete analysis since the extension is rather straightforward.

\(^4\)The literature of “sequential voting” is similar to our study in that voters observe votes of previous voters (Dekel and Piccione, 2000; Callander, 2007). However, these studies focus on one round of voting in which voters make decisions sequentially, while the current paper allows multiple rounds of votes.
jurors are excluded from information of previous trials or signals of guilt and innocence have exactly identical accuracy. Thus the current work provides a qualification on results of Coughlan (2000) and other works on mistrials in light of our fully dynamic setting.

More broadly, the current work is part of research on jury design with a special attention to the unanimity rule. The seminal contribution of Feddersen and Pesendorfer (1998) was extended to models of voting with continuum signals (Duggan and Martinelli, 2001; Meirowitz, 2002) and private information about preferences (Gerardi, 2000). Coughlan (2000) introduces a model in which jurors deliberate by taking a straw vote before the final vote, and shows that informative voting is an equilibrium for a nontrivial range of parameters. Deliberation in juries and comittees attracted much attention in subsequent works (Doraszelski et al., 2003; Gerardi and Yariv, 2007a; Austen-Smith and Feddersen, 2005). Particularly related to this paper is a paper by Austen-Smith and Feddersen (2006), who study voting rules when deliberation is allowed and jurors are uncertain about preferences of others. They show that informative voting is not an equilibrium under the unanimity rule if people have minimal heterogeneity in preferences, while informative voting may be an equilibrium under nonunanimous rules. Our study and theirs can be seen as complementing each other, casting doubt on the efficacy of the unanimity rule in a realistic model of jury decision.

Finally, the current work contributes to the modern literature of jury design under strategic voting pioneered by Austen-Smith and Banks (1996). They study the strategic aspect of jury decision and show that informative voting often fails to be an equilibrium. McLennan (1998) and Wit (1998) show that there exists an equilibrium, which may not be informative, that aggregates jurors' information in an adequate manner. Recent study of costly information acquisition (Persico, 2004; Gerardi and Yariv, 2007b) again poses an important qualification to the original assertion of Condorcet (1976) that jury decision improves as the number of jurors increases. Study of Condorcet Jury theorems has long tradition in a more statistically oriented literature (Black, 1958; Grofman and Feld, 1988; Young, 1988; Berg, 1993; Ladha, 1992, 1993).

2 The Basic Model

There are \( n \) jurors, \( N = \{1, 2, \ldots, n\} \), who vote to decide the fate of the defendant in a criminal charge. There are two states of the world \( \omega \in \{G, I\} \): the defendant is either guilty (denoted \( G \)) or innocent (\( I \)). The prior probabilities that the state is \( G \) and \( I \) are \( r \in (0, 1) \) and \( 1-r \), respectively.

There are two possible outcomes for the jury decision: The defendant is convicted (denoted \( C \)) or acquitted (\( A \)). Each juror votes either for conviction or acquittal. A voting rule is described by an integer \( \hat{k} \in (n/2, n] \). If at least \( \hat{k} \) jurors vote for conviction, then the defendant is convicted; if at least \( \hat{k} \) jurors vote for acquittal, then the defendant is acquitted; if neither of these happens, then a mistrial is declared. When a mistrial is declared, a new jury \( N' \) composed of \( n \) members is formed and the same procedure as described above proceeds, and so on. In general, we denote the jury after \( m \) mistrials by \( N^m \). Hence the first jury can be written as \( N^0 \), the jury after one mistrial as \( N^1 \), and so on.

Given the state of the world \( \omega \in \{G, I\} \), each juror receives a noisy signal of the state of the world. Let \( s_j \) denote the signal received by juror \( j \). There are two possible signals, \( g \) or \( i \). For every \( j \), we assume

\[
\begin{align*}
\text{Prob}(s_j = g | \omega = G) &= p_g \in (1/2, 1), \\
\text{Prob}(s_j = i | \omega = I) &= p_i \in (1/2, 1).
\end{align*}
\]

In words, \( p_g \) and \( p_i \) represent the probabilities that a juror observes the correct signal when the true states are \( g \) and \( i \), respectively.

We say that the outcome of the trial is correct if either the defendant is guilty and convicted or she is innocent and acquitted. The utility of juror \( j \) when the state is \( \omega \) and the decision \( d \in \{C, A\} \) is made is denoted by \( u_j(d, \omega) \), and defined as

\[
\begin{align*}
u_j(C, G) &= u_j(A, I) = 0, \\
u_j(C, I) &= -q_j, \\
u_j(A, G) &= -(1-q_j),
\end{align*}
\]

where \( q_j \in (0, 1) \). Juror \( j \) prefers conviction to acquittal if and only if she places at least probability \( q_j \) that the defendant is guilty. We assume that

\[
\begin{align*}
\bar{q} &\equiv \sup_{j \in \bigcup_{m=0}^{\infty} N^m} q_j < 1, \\
\underline{q} &\equiv \inf_{j \in \bigcup_{m=0}^{\infty} N^m} q_j > 0.
\end{align*}
\]
In words, while we allow for an infinite number of potential jurors, levels of “reasonable doubt” for the population is bounded away from the extreme values, 0 and 1.\(^6\)

As there may be several trials before the jury decision is finalized in our model, we extend the utility of each juror as follows. When a decision \(d\) is finally made, utility of juror \(j\) is given by \(u_j(d, \omega)\). When an infinite sequence of mistrials occurs, each juror receives some fixed utility.\(^7\)

A strategy of a juror is a rule that, based on all the available information from previous trials (if any) and the signal she observes, votes for either \(C\) or \(A\). A voting strategy of \(j\) is informative if the strategy chooses \(C\) if \(s_j = g\) and \(A\) if \(s_j = i\) irrespective of any available information other than the signal \(s_j\). A strategy profile is informative if strategies of all the jurors are informative. We say that informative voting is an equilibrium if it is a sequential equilibrium for every juror to take an informative strategy.

### 3 Results

We assume two different informational structures.

#### 3.1 Informative Voting without Knowledge of Numerical Split

We assume that all jurors in the \((m+1)\)st trial, \(N^m\), know the fact that all previous trials resulted in mistrials, but they do not know the numerical split, that is, how many votes were cast for conviction in previous trials (let alone which juror voted for conviction). In order to focus on the implication of mistrials, we assume that there is at least one numerical split under which a mistrial is declared. This assumption is a mild restriction on the model: If \(n\) is even, then it is no restriction except the maintained assumption \(k > n/2\); if

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\(^6\)This assumption is satisfied quite broadly. One example is a model in which the parameter \(q_j\) is chosen from a finite set of possible values in \((0, 1)\). A stationary case in which the utility characteristic of each jury is the same for every period is a particular case of such a specification. Even if the population of potential jurors does not satisfy the bound, jury selection procedures (often called voir dire) may render the bound plausible, since the processes prevent highly biased individuals from serving in juries.

\(^7\)Since our analysis focuses on informative voting, infinite mistrials occur with probability zero. Hence no specific assumption on the utility for infinite mistrials is important for the analysis. Also we note that retrials may be costly and prosecutors may decide not to seek a retrial after a mistrial with some probability in reality, but our results carry over when we introduce these additional complications.
Let $m_j^\omega$ be the expected disutility of juror $j$ when a mistrial is declared at the jury in which $j$ votes, when each juror follows an informative voting and the true state is $\omega$. For different states, such values are calculated by the following recursive formulae,

$$m_j^G = (1 - q_j)\text{Prob}(A|G) + m_j^G\text{Prob}(M|G),$$
$$m_j^I = q_j\text{Prob}(C|I) + m_j^I\text{Prob}(M|I),$$

where $\text{Prob}(A|\omega)$, $\text{Prob}(C|\omega)$ and $\text{Prob}(M|\omega)$ are, respectively, the probabilities that a single jury decides acquittal, conviction and mistrial under informative voting when the true state is $\omega$.

Let $\text{Prob}(\omega, |s|j = \hat{k} - 1|s_j, m)$ be the probability that the state is $\omega$ and $\hat{k} - 1$ jurors out of $n - 1$ observe signal $s$, conditional on the event that $j \in N^m$ observes signal $s_j$ and $m$ mistrials before she gets to vote. Define, for juror $j \in N^m$,

$$\Delta EU_j(C - A|s_j) \equiv -q_j\text{Prob}(I, |g|j = \hat{k} - 1|s_j, m) - m_j^G\text{Prob}(G, |i|j = \hat{k} - 1|s_j, m) - m_j^I\text{Prob}(I, |i|j = \hat{k} - 1|s_j, m) - [-u_j\text{Prob}(G, |i|j = \hat{k} - 1|s_j, m) - m_j^I\text{Prob}(I, |g|j = \hat{k} - 1|s_j, m)].$$

In words, $\Delta EU_j(C - A|s_j)$ is the relative expected utility for juror $j \in N^m$ of voting for conviction rather than for acquittal, conditional on that $j$ gets to vote (that is, $m$ mistrials have been declared so far) and she sees signal $s_j$.

Informative voting is an equilibrium if and only if

$$\Delta EU_j(C - A|g) \geq 0,$$
$$\Delta EU_j(C - A|i) \leq 0,$$

for any $m \in \mathbb{N}$ and $j \in N^m.$

Conditional probabilities $\text{Prob}(\omega, |s|j = \hat{k} - 1|s_j, m)$ can be computed explicitly. For example,

$$\text{Prob}(G, |g|j = \hat{k} - 1|g, m) = \frac{r\binom{n-1}{\hat{k}}(1 - \text{Prob}(M|G))m^{\hat{k}}(1 - p_g)^{n-k}}{r\binom{n-1}{\hat{k}}(1 - \text{Prob}(M|G))m^{\hat{k}}(1 - p_g)^{n-k} + (1 - r)(\text{Prob}(M|I))m(1 - p_i)}.$$

The following lemma is key for our analysis.
Lemma 1. $\text{Prob}_s(M|G)$ and $\text{Prob}_s(M|I)$ are strictly decreasing in $p_g$ and $p_i$, respectively. $p_g = p_i$ implies $\text{Prob}_s(M|G) = \text{Prob}_s(M|I)$, $p_g > p_i$ implies $\text{Prob}_s(M|G) < \text{Prob}_s(M|I)$, and $p_g < p_i$ implies $\text{Prob}_s(M|G) > \text{Prob}_s(M|I)$.

Proof. Suppose that $\omega = G$. Since a mistrial occurs if and only if the number of votes for conviction is in $\{ n - \hat{k} + 1, n - \hat{k} + 2, \ldots, \hat{k} - 1 \}$ in each trial, we obtain

$$\text{Prob}_s(M|G) = \sum_{k = n - \hat{k} + 1}^{\hat{k} - 1} \binom{n}{k} p_g^k (1 - p_g)^{n-k}.$$ 

Differentiating the above expression with respect to $p_g$, we obtain

$$\frac{d(\text{Prob}_s(M|G))}{dp_g} = \sum_{k = n - \hat{k} + 1}^{\hat{k} - 1} \binom{n}{k} k p_g^{k-1} (1 - p_g)^{n-k} - \sum_{k = n - \hat{k} + 1}^{\hat{k} - 1} \binom{n}{k} (n-k) p_g^k (1 - p_g)^{n-k-1}$$

$$= \sum_{k = n - \hat{k}}^{\hat{k} - 2} \binom{n}{k+1} (k+1) p_g^k (1 - p_g)^{n-k-1} - \sum_{k = n - \hat{k} + 1}^{\hat{k} - 1} \binom{n}{k} (n-k) p_g^k (1 - p_g)^{n-k-1}$$

$$= \sum_{k = n - \hat{k} + 1}^{\hat{k} - 2} \left( \binom{n}{k+1} (k+1) - \binom{n}{k} (n-k) \right) p_g^k (1 - p_g)^{n-k-1}$$

$$+ \binom{n}{n - \hat{k} + 1} (n - \hat{k} + 1) p_g^{n-k} (1 - p_g)^{\hat{k} - 1} - \binom{n}{\hat{k} - 1} (n - \hat{k} + 1) p_g^{k-1} (1 - p_g)^{n-k}.$$ 

($\#$)

In general the following equality holds:

$$\binom{n}{k+1} (k+1) = \frac{n!}{(k+1)! (n-k-1)!} \times (k+1)$$

$$= \frac{n!}{k!(n-k-1)!}$$

$$= \frac{n!}{k!(n-k)!} \times (n-k)$$

$$= \binom{n}{k} (n-k).$$

Therefore the terms in summation in equation ($\#$) cancel out with one another. This and the fact $\binom{n}{n - k + 1} = \binom{n}{k - 1}$ give us a simplified expression of...
\[
\frac{d(\text{Prob}_s(M|G))}{dp_g} = \left(\begin{array}{c} n \\ n - \hat{k} + 1 \end{array}\right) (n - \hat{k} + 1)p_g^{n-\hat{k}}(1-p_g)^{\hat{k}-1} - \left(\begin{array}{c} n \\ \hat{k} - 1 \end{array}\right) (n - \hat{k} + 1)p_g^{\hat{k}-1}(1-p_g)^{n-\hat{k}} \\
= \left(\begin{array}{c} n \\ n - \hat{k} + 1 \end{array}\right) (n - \hat{k} + 1)|p_g^{n-\hat{k}}(1-p_g)^{\hat{k}-1} - p_g^{\hat{k}-1}(1-p_g)^{n-\hat{k}}|.
\]

Since the assumption \( \hat{k} > (n+1)/2 \) implies \( \hat{k} - 1 > n - \hat{k} \) and the assumption \( p_g \in (1/2, 1) \) implies \( p_g > 1 - p_g \), the last expression is always negative. Therefore \( \text{Prob}_s(M|G) \) is strictly decreasing in \( p_g \) over interval \((1/2, 1)\). A similar argument establishes that \( \text{Prob}_s(M|I) \) is strictly decreasing in \( p_i \).

Since

\[
\text{Prob}_s(M|I) = \sum_{k=n-\hat{k}+1}^{\hat{k}-1} \left(\begin{array}{c} n \\ k \end{array}\right) p_i^k (1-p_i)^{n-k}
\]

has the same functional form as \( \text{Prob}_s(M|G) \) with \( p_g \) replaced by \( p_i \), clearly \( p_g = p_i \) implies \( \text{Prob}_s(M|G) = \text{Prob}_s(M|I) \). Finally, \( p_g > p_i \) implies \( \text{Prob}_s(M|G) < \text{Prob}_s(M|I) \) and \( p_g < p_i \) implies \( \text{Prob}_s(M|G) > \text{Prob}_s(M|I) \) since \( \text{Prob}_s(M|G) \) and \( \text{Prob}_s(M|I) \) are decreasing by the earlier part of the claim.

The main part of Lemma 1 states that the noisier the signal, the higher the probability that a hung jury occurs. Intuitively, if the signal accuracy decreases, then jurors become more likely to receive mistaken signals and as a result become unlikely to agree on the right verdict. The formal proof is more complicated since a noisy signal increases the probability that the jury agrees on the wrong decision, offsetting the first effect. The proof establishes that the first effect dominates the second.

Equipped with the above lemma, we first consider a special case in which the signals for guilt and innocence are equally accurate, that is, \( p_g = p_i = p \) for some \( p \in (1/2, 1) \).

**Proposition 1.** Suppose \( p_g = p_i = p \in (1/2, 1) \). Under this condition, informative voting forms an equilibrium if and only if, for each juror \( j \),

\[
\frac{r}{1-r} \cdot \frac{1-p}{p} \leq \frac{q_j}{1-q_j} \leq \frac{r}{1-r} \cdot \frac{p}{1-p}.
\]

**Proof.** By Lemma 1, \( p_g = p_i = p \) implies \( \text{Prob}_s(M|G) = \text{Prob}_s(M|I) \). Therefore \( \text{Prob}(G, |g|_j = \hat{k} - 1|s_j, m) \) can be simplified as

\[
\text{Prob}(G, |g|_j = \hat{k} - 1|g, m) = \left(\begin{array}{c} n - 1 \\ \hat{k} - 1 \end{array}\right) \frac{rp^k(1-p)^{n-k}}{rp + (1-r)(1-p)}.
\]
The above expression is independent of \( m \), and coincides with the expression in the proof of Proposition 1 of Coughlan (2000). Similarly \( \text{Prob}(\omega, |s|_{j} = \hat{k} - 1|s_{j}, m) \) is independent of \( m \) and coincides with the expression in Coughlan (2000) for any \( \omega \in \{G, I\} \) and \( s, s_{j} \in \{g, i\} \). Therefore the sequential rationality condition for a juror in any \( N^{m} \) is given by the same expression as his, and by the same argument as his Proposition 1, we obtain the necessary and sufficient condition presented in the statement of the proposition. \( \square \)

This proposition verifies that Proposition 1 of Coughlan (2000) derived in a reduced form continues to hold in our explicitly dynamic setting, if we assume that signals for guilt and innocence have exactly identical accuracy.

Next, we consider a generic case in which the signal accuracies are not exactly identical between guilt and innocence, that is, \( p_{g} \neq p_{i} \). There seems to be no convincing reason that signal accuracies for guilt and innocence are exactly equal, so \( p_{g} \neq p_{i} \) is probably a typical situation in real application.

**Theorem 1.** Suppose \( p_{g} \neq p_{i} \). Then informative voting never forms an equilibrium.

**Proof.** First consider the case with \( p_{g} > p_{i} \). By Lemma 1, \( p_{g} > p_{i} \) implies \( \text{Prob}_{s}(M|G) < \text{Prob}_{s}(M|I) \). Therefore \( \text{Prob}(G, |s|_{j} = \hat{k} - 1|s_{j}, m) \to 0 \) as \( m \to \infty \) for each pair of \( s, s_{j} \in \{g, i\} \). Moreover we have \( \text{Prob}(I, |g|_{j} = \hat{k} - 1|s_{j}, m) \to \left( \frac{n-1}{k-1} \right) (1 - p_{i})^{k-1} \) and \( \text{Prob}(I, |i|_{j} = \hat{k} - 1|s_{j}, m) \to \left( \frac{n-1}{k-1} \right) p_{i}^{k-1} \) for \( s_{j} \in \{g, i\} \) as \( m \to \infty \).

Juror \( j \in N^{m} \) who observes \( s_{j} = g \) has incentives to vote for conviction if and only if \( \Delta EU_{j}(C - A|g) \geq 0 \). The expression is

\[
\Delta EU_{j}(C - A|g) = -q_{j} \text{Prob}(I, |g|_{j} = \hat{k} - 1|g, m) - m_{j}^{G} \text{Prob}(G, |i|_{j} = \hat{k} - 1|g, m)
- m_{j}^{I} \text{Prob}(I, |i|_{j} = \hat{k} - 1|g, m)
- [-\alpha q_{j} \text{Prob}(G, |i|_{j} = \hat{k} - 1|g, m) - m_{j}^{G} \text{Prob}(G, |g|_{j} = \hat{k} - 1|g, m)]
\leq -q \text{Prob}(I, |g|_{j} = \hat{k} - 1|g, m) - m_{j}^{G} \text{Prob}(G, |i|_{j} = \hat{k} - 1|g, m)
- m_{j}^{I} \text{Prob}(I, |i|_{j} = \hat{k} - 1|g, m)
- [-\alpha q_{j} \text{Prob}(G, |i|_{j} = \hat{k} - 1|g, m) - m_{j}^{G} \text{Prob}(G, |g|_{j} = \hat{k} - 1|g, m)]
- m_{j}^{I} \text{Prob}(I, |g|_{j} = \hat{k} - 1|g, m)]
\]

Since \( \lim_{m \to \infty} \text{Prob}(I, |g|_{j} = \hat{k} - 1|g, m) = \left( \frac{n-1}{k-1} \right) (1 - p_{i})^{k-1} < \left( \frac{n-1}{k-1} \right) p_{i}^{k-1} = \lim_{m \to \infty} \text{Prob}(I, |i|_{j} = \hat{k} - 1|g, m) \) from the earlier argument (the inequality resulting from the assumption \( p_{i} > 1/2 \)), we obtain \( \lim_{m \to \infty} \Delta EU_{j}(C - A|g) \geq 0 \).

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8The expression is denoted by \( \text{Prob}(G \cap |C|_{j} = \hat{k} - 1|s_{j} = g) \) in Coughlan (2000).
\[ A|g) \leq \left(\frac{n-1}{k-1}\right) (1-p_i)^{k-1} q < 0. \] Thus for any sufficiently large \( m \), \( \Delta EU_j(C - A|g) < 0 \) for any juror \( j \in N^m \). In other words, all jurors in \( N^m \) have strict incentives to vote for acquittal even if they observe private signal \( g \), which establishes that informative voting is not an equilibrium.

In the case with \( p_g < p_i \), an argument analogous to the above one establishes that, for any sufficiently large \( m \), all jurors in \( N^m \) who observe private signal \( i \) have strict incentives to vote for conviction. Hence informative voting does not form an equilibrium in this case either.

Theorem 1 is intuitive in light of Lemma 1. Suppose, for instance, that the signal of guilt is more accurate than that of innocence. Then, jurors who vote informatively are more likely to result in a hung jury when the defendant is innocent than when she is guilty by Lemma 1. Hence, if a juror knows that there was a hung jury before the current trial, she infers that the defendant is more likely to be innocent by Bayes’ law. If hung juries occur repeatedly, information from previous trials will become so strong that a juror is willing to vote for acquittal even if she observes a guilty signal. Thus informative voting fails to be an equilibrium.

Remark 1. Based on the fact that the inequality in Proposition 1 becomes less demanding when \( p \) becomes larger, Coughlan (2000) argues that improving the accuracy of signals renders informative voting “more likely.” Theorem 1 sheds new light on the assertion. Suppose that originally the accuracy of both signals are identical. Then informative voting is an equilibrium for some utilities of jurors by Proposition 1. If the accuracy of only one of the signals improves, then Theorem 1 implies that informative voting is not an equilibrium any more. Thus, improvement of signal accuracy sometimes diminishes the prospect of informative voting.

3.2 Informative Voting with Knowledge of Numerical Split

In this section, we assume that all jurors in the \((m+1)\)st trial, \( N^m \), know the fact that all previous trials resulted in mistrials, and moreover how many votes were cast for conviction and acquittal, respectively, in each of previous trials. We assume that there are at least two numbers \( \tilde{k}, \tilde{k} + 1 \) of votes for conviction under which a mistrial is declared. This assumption is well motivated in the current context: Otherwise, a juror faced with a mistrial can correctly infer the number of votes for conviction, and there is no interesting difference between the models of this section and the last section. This restriction is equivalent to assuming \( \tilde{k} \geq n/2 + 2 \).
Under the current informational structure, the negative result is more striking than in the last section. More specifically, for any parameter value, informative voting fails to be an equilibrium.

**Theorem 2.** Informative voting never forms an equilibrium.

**Proof.** Let \( \tilde{k} \) and \( \tilde{k} + 1 \) be numbers of votes for conviction that result in a mistrial in the current voting rule. Let \( \text{Prob}_s(k|\omega) \) be the probability that a single jury results in \( k \) votes for conviction under informative voting when the true state is \( \omega \). First we show that \( \text{Prob}_s(\tilde{k}|G) \neq \text{Prob}_s(\tilde{k}|I) \) or \( \text{Prob}_s(\tilde{k}+1|G) \neq \text{Prob}_s(\tilde{k}+1|I) \). Indeed, suppose \( \text{Prob}_s(\tilde{k}+1|G) = \text{Prob}_s(\tilde{k}+1|I) \). This implies that

\[
\text{Prob}_s(\tilde{k}|G) = \frac{1 - p_g}{p_g} \binom{n}{\tilde{k}} \binom{n}{\tilde{k}+1}(1 - p_g)^{n-\tilde{k}-1}
\]

\[
= \frac{1 - p_g}{p_g} \binom{n}{\tilde{k}} \text{Prob}_s(\tilde{k}+1|G)
\]

\[
= \frac{1 - p_g}{p_g} \binom{n}{\tilde{k}} \text{Prob}_s(\tilde{k}+1|I)
\]

\[
< \frac{p_i}{1 - p_i} \binom{n}{\tilde{k}} \text{Prob}_s(\tilde{k}+1|I)
\]

\[
= \frac{p_i}{1 - p_i} \binom{n}{\tilde{k}} \binom{n}{\tilde{k}+1}(1 - p_g)^{\tilde{k}+1} p_i^{n-\tilde{k}-1}
\]

\[
= \binom{n}{\tilde{k}} (1 - p_i) \tilde{k} p_i^{n-\tilde{k}}
\]

where the inequality holds because of assumptions \( p_i, p_g \in (1/2, 1) \).

Given the above argument, there exists \( k \) such that \( \text{Prob}_s(k|G) \neq \text{Prob}_s(k|I) \). Let us assume that \( \text{Prob}_s(k|G) < \text{Prob}_s(k|I) \) (the case with \( \text{Prob}_s(k|G) > \text{Prob}_s(k|I) \) can be shown analogously.) Consider a sequence of \( m \) trials such that at each of them, \( k \) votes are cast for conviction. Then, by an argument analogous to the final part of the proof of Theorem 1, for any sufficiently large \( m \), all jurors in \( N^m \) have strict incentives to vote for acquittal even if they observe signal \( g \). Hence informative voting is not an equilibrium. \( \square \)

Proposition 1, Theorem 1 and Theorem 2 demonstrate that informative voting may be more difficult when numerical split is known to subsequent
juries. More generally, our results and those in Coughlan (2000) suggest that informative voting may become more difficult when more information becomes available to jurors. This observation may be surprising, as “more informed” jurors may actually fail to vote informatively rather than the less informed ones. This observation may support some legal practice such as (1) jury selection procedures (voir dire) to select individuals with little prior knowledge of the case, and (2) sequestering of jurors during their duty that prevents them from acquiring information about the case by reading newspaper articles or watching televisions.

**Remark 2.** Our analysis assumes that all jurors know the existence of previous trials (in Section 3.1) and numerical split (in Section 3.2). When jurors are informed through the mass media, it is likely that only some of the jurors have such information. All our results can be extended to such cases. More specifically, proofs of Theorem 1 and Theorem 2 carry over with little modification as long as there is a juror with such knowledge in a sufficiently late round of the trials. Proposition 1 extends even more broadly, whether or not there is any informed juror.

### 4 Concluding Remarks

We studied jury decision in an explicitly dynamic environment with mistrials and retrials. We find that informative voting is rarely an equilibrium. Thus the assertion of Coughlan (2000) that mistrials help the unanimity rule to perform well is in question. As such, the current paper contributes to the debate on the desirability of the unanimity rule and jury design more generally (Austen-Smith and Banks, 1996; Feddersen and Pesendorfer, 1998). Our analysis also provides theoretical support for a wide use of jury selection and sequestering. These measures promote informative voting by keeping jurors uninformed about previous trials of the case.

Our impossibility results (Theorems 1 and 2) are proven by showing that jurors in later rounds have incentives to ignore their private signals and engage in noninformative voting given other jurors vote informatively. This proof should not be taken, however, to suggest that jurors in early rounds of trials vote informatively. As jurors in later rounds have incentives to vote strategically, incentives of jurors in early rounds may be altered through anticipation of what verdicts are likely to be made if a mistrial happens. Thus even jurors in early rounds of trials may have incentives to engage in noninformative and strategic voting.

A natural next step is equilibrium analysis. An obvious question is what class of equilibria should be regarded as likely outcomes in the current con-
text of dynamic voting. As in most voting models, there may be a myriad of equilibria, some of which are not interesting: For example, a trivial voting strategy profile where every juror ignores all information and votes for conviction is an equilibrium for any nonunanimous rule. A satisfactory analysis of voting equilibria under different voting rules with mistrials is beyond the scope of this paper and left for future research.

References


